Representation Theory of Finite Groups

Midterm Examination

September 12, 2024

Instructions: All questions carry ten marks. Vector spaces and representations are assumed to be finite dimensional over the field of complex numbers.

1. Let m and n be two natural numbers. Determine the tensor product $(\mathbb{Z}/m\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Z}/n\mathbb{Z})$.
2. Let G be a nonabelian group of order 55. Determine the class equation of G and the number of elements in each of its counjugacy classes.
3. Let V be an irreducible representation of a finite group G. Show that, up to scalars, there is a unique Hermitian inner product on V that is 'preserved' by G.
4. Give an example of a nonabelian finite group G that does not have any faithful irreducible (finite dimensional) representation over $\mathbb{C}$. Justify your answer.
5. Let H be the subgroup of order 11 in a nonabelian group G of order 55. Let W be the one dimensional representation of H corresponding to a primitive 11^{th} root of unity.

   Prove or disprove: $\mathrm{Ind}_{\mathrm{H}}^{\mathrm{G}}(\mathrm{W})$ is an irreducible representation of G.